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Gauge fluctuations in superconducting films

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Abstract. In this paper we consider a type-I superconducting film modeled by the Ginzburg-Landau model, confined between two parallel planes a distance L apart from one another. Our approach is based on the Gaussian effective potential in the transverse unitarity gauge, which allows to treat gauge contributions in a compact form. Using techniques from dimensional and ζ-function regularizations, modified by the confinement conditions, we investigate the critical temperature as a function of the film thickness L. The contributions from the scalar self-interaction and from the gauge fluctuations are clearly identified. The model suggests the existence of a minimal critical thickness below which superconductivity is suppressed. A comparison with present experimental observations is done.

PACS. 74.20.-z Theories and models of superconducting state – 11.15.Ex Spontaneous breaking of gauge symmetries – 05.10.Cc Renormalization group methods

1 Introduction

It is currently assumed to be a good approximation to neglect magnetic thermal fluctuations in the Ginzburg-Landau (GL) model, to explore general features of superconducting transitions. However, this approximation excludes the study of the so called charged phase transitions. They only can be investigated when fluctuations of the gauge field are taken into account, which make explicitly appear in the thermodynamic quantities the coupling constant for the interaction between the scalar field and the gauge field (the charge of the boson). This is a hard problem to be directly faced and many attempts have been done to go beyond the pioneering work of reference [1]. In what concerns physical situations already present in the literature, a large amount of work has been done on the GL model applied to the study of superconductors, both in the single component and in the N-component versions of the model, using the renormalization group approach. The interested reader can find an account on the state of the subject for both type-I and type-II superconductors and related topics in references [2–7].

Also, in the last three decades both theoretical and experimental works have been done on the superconducting properties of thin films [8–20]. An interesting result emerging from these works is that the superconducting transition temperature T_c is reduced as the film thickness is decreased.

Besides, effects of spatial boundaries on the behaviour of physical systems appear in several forms in the literature. At the level of effective field theories, in many cases, boundaries can be modeled by considering for instance a Dirac fermionic field whose mass changes sign as it crosses the defect, which means that the domain wall can be interpreted as a kind of a critical boundary [21,22]. Questions concerning stability and the existence of phase transitions may also be raised if we enquire about the behaviour of field theories as function of spatial boundaries. The existence of phase transitions would be, in this case, also associated to some spatial parameters describing the breaking of translational invariance, in our case the distance L between planes confining the system (a superconducting film of thickness L). In particular the question of how the superconducting critical temperature could depend on the thickness of the film can be raised.

In this paper, we intend to study superconducting films within a field theory framework. We consider the GL theory, the system being submitted to the constraint of confinement between two parallel planes a distance L apart from one another. From a physical point of view, for dimension $d = 3$ and introducing temperature by means of the mass term in the Hamiltonian, this corresponds to a film-like material. We investigate the behaviour of the system taking into account gauge fluctuations, which means that charged transitions are included in our work. We are particularly interested in the problem of how the critical behaviour depends on the film thickness L. This study is done by means of the Gaussian Effective Potential (GEP) as developed in references [23–27], together with a spatial compactification mechanism introduced in recent publications [28,29].

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In the next section, we apply the functional approach of the Gaussian Effective Potential formalism to the Ginzburg-Landau model, obtaining the mass, which obeys a generalized "Gaussian" Dyson-Schwinger equation. In Section 3, we extend the formalism of Section 2 to the GL model confined between two parallel planes, and we study the critical behaviour of the system. In particular, we obtain an expression for the critical temperature as a function of the spacing between the planes (the film thickness). A qualitative and quantitative comparison of our results with experimental data is done in Section 4. Finally, we summarize our results in Section 5.

Unless explicitly stated we use all along the paper Natural Units (NU), $\hbar = c = k_B = 1$. Also our nomenclature, notations and conventions are those usual in the quantum field theoretical approach to critical phenomena [2–7]. These choices are not the usual ones in solid state physics, but are the more appropriate for our purposes.

2 The Gaussian effective potential for the Ginzburg-Landau model

We begin by briefly presenting the study of the U(1) Scalar Electrodynamics in the transverse unitarity gauge, along the lines developed in reference [27]. We start from the Hamiltonian density of the GL model in Euclidean d-dimensional space (recall we are in NU) written in the form [30],

$$\mathcal{H}' = \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}|(\partial_\mu - ieA_\mu)\Psi|^2 + \frac{1}{2}m_0^2|\Psi|^2 + \lambda(|\Psi|^2)^2, \quad (1)$$

where Ψ is a complex field, and m_0 is the bare mass. The components of the transverse magnetic field, $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ $(\mu, \nu = 1, ..., d)$ are related to the d-dimensional potential vector by the well known equation,

$$\frac{1}{2}F_{\mu\nu}F^{\mu\nu} = |\nabla \times \mathbf{A}|^2. \quad (2)$$

In order to obtain only physical degrees of freedom, we can introduce two real fields instead of the complex field Ψ, assuming a transverse unitarity gauge. We can define the field in terms of two real fields, as $\Psi = \phi e^{i\gamma}$, together with the gauge transformation $\mathbf{A} \to \mathbf{A} - 1/e\nabla\gamma$. The unitarity gauge makes the original transverse field to acquire a longitudinal component $\mathbf{A}_L$ proportional to $\nabla\gamma$. Then the original functional integration over Ψ and Ψ^* in the generating functional of correlation functions, becomes an integration over ϕ, $\mathbf{A}_T$ and $\mathbf{A}_L$. The longitudinal component of the vector potential can be integrated out, leading to the generating functional (up to constant terms),

$$Z[j] = \int D\phi\, DA_T \exp\left[-\int d^dx\mathcal{H} + \int d^dx\, j\phi\right], \quad (3)$$

where the Hamiltonian is

$$\mathcal{H} = \frac{1}{2}(\nabla\phi)^2 + \frac{1}{2}m_0^2\phi^2 + \lambda\phi^4 + \frac{1}{2}e^2\phi^2A^2 + \frac{1}{2}(\nabla \times \mathbf{A})^2 + \frac{1}{2\epsilon}(\nabla \cdot \mathbf{A})^2. \quad (4)$$

We have introduced above a gauge fixing term, the limit $\epsilon \to 0$ being taken later on after the calculations have been done. In equation (4) and in what follows, unless explicitly stated, $\mathbf{A}$ stands for the *transverse* gauge field.

The Gaussian effective potential can be obtained from equation (4), performing a shift in the scalar field in the form $\phi = \tilde{\phi} + \varphi$, which allows to write the Hamiltonian in the form

$$\mathcal{H} = \mathcal{H}_0 + \mathcal{H}_{int}, \quad (5)$$

with $\mathcal{H}_0$ being the free part and $\mathcal{H}_{int}$ the interaction part, given respectively by

$$\mathcal{H}_0 = \left[\frac{1}{2}(\nabla\tilde{\phi})^2 + \frac{1}{2}\Omega^2\tilde{\phi}^2\right] + \left[\frac{1}{2}(\nabla \times \mathbf{A})^2 + \frac{1}{2}\Delta^2 A_\mu A^\mu + \frac{1}{2\epsilon}(\nabla \cdot \mathbf{A})^2\right], \quad (6)$$

and

$$\mathcal{H}_{int} = \sum_{n=0}^{4} v_n\tilde{\phi}^n + \frac{1}{2}\left(e^2\varphi^2 - \Delta^2\right)A_\mu A^\mu + \frac{1}{2}e^2\tilde{\phi}A_\mu A^\mu\varphi + \frac{1}{2}e^2A_\mu A^\mu\varphi^2, \quad (7)$$

where

$$v_0 = \frac{1}{2}m_0^2\varphi^2 + \lambda\varphi^4, \quad (8)$$

$$v_1 = m_0^2\varphi + 4\lambda\varphi^3, \quad (9)$$

$$v_2 = \frac{1}{2}m_0^2\varphi^2 + 6\lambda\varphi^2 - \frac{1}{2}\Omega^2, \quad (10)$$

$$v_3 = 4\lambda\varphi, \quad (11)$$

$$v_4 = \lambda. \quad (12)$$

It is clear from equations (5, 6) and (7), that $\mathcal{H}$ describes two interacting fields, a real scalar field ϕ of mass Ω and a real vector gauge field $\mathbf{A}$ of mass Δ.

The effective potential, which is defined by

$$V_{eff}[\varphi] = \frac{1}{V}\left[-\ln Z + \int d^dxj\varphi\right], \quad (13)$$

where V is the total volume, can be obtained at first order from standard methods from perturbation theory. One can find, from equations (3, 6) and (7),

$$V_{eff}[\varphi] = I_1^d(\Omega) + 2I_1^d(\Delta) + \frac{1}{2}m_0^2\varphi^2 + \lambda\varphi^4 + \frac{1}{2}\left[m_0^2 - \Omega^2 + 12\lambda\varphi^2 + 6\lambda I_0^d(\Omega)\right]I_0^d(\Omega) + \left[e^2\left(\varphi^2 + I_0^d(\Omega)\right) - \Delta^2\right]I_0^d(\Delta), \quad (14)$$

where,

$$I_0^d(M) = \int \frac{d^dk}{(2\pi)^d}\frac{1}{k^2+M^2}, \tag{15}$$

$$I_1^d(M) = \frac{1}{2}\int \frac{d^dk}{(2\pi)^d}\ln(k^2+M^2), \tag{16}$$

with $k = (k_1, ..., k_d)$ being the d-dimensional momentum.

The Gaussian effective potential is derived by the requirement that $V_{eff}[\varphi]$ must be stationary under variations of the masses Δ and Ω. This means that values $\overline{\Omega}$ and $\overline{\Delta}$ for the masses Ω and Δ should be found such that the conditions,

$$\left.\frac{\partial V_{eff}}{\partial \Omega^2}\right|_{\Omega^2=\overline{\Omega}^2} = 0, \tag{17}$$

$$\left.\frac{\partial V_{eff}}{\partial \Delta^2}\right|_{\Delta^2=\overline{\Delta}^2} = 0, \tag{18}$$

be simultaneously satisfied. These conditions generate the gap equations,

$$\overline{\Omega}^2 = m_0^2 + 12\lambda\varphi^2 + 12\lambda I_0^d(\overline{\Omega}) + 2e^2 I_0^d(\overline{\Delta}), \tag{19}$$

$$\overline{\Delta}^2 = e^2\varphi^2 + e^2 I_0^d(\overline{\Omega}). \tag{20}$$

Replacing Ω and Δ in equation (14) by the solutions $\overline{\Omega}$ and $\overline{\Delta}$, of equations (19, 20) we obtain for the GEP the formal expression,

$$\overline{V}_{eff}[\varphi] = I_1^d(\overline{\Omega}) + 2I_1^d(\overline{\Delta}) + \frac{1}{2}m_0^2\varphi^2 + \lambda\varphi^4 - 3\lambda[I_0^d(\overline{\Omega})]^2 - e^2 I_0^d(\overline{\Omega})I_0^d(\overline{\Delta}). \tag{21}$$

Notice that equations (19, 20) are a pair of coupled equations, which can be solved by numerical methods. Nevertheless we do not need to go through the numeric solution as we are interested in the limit of criticality.

Next we intend to write an expression for the Gaussian mass, $\overline{m}$, obtained in our case from the standard prescription, as the second derivative of the *Gaussian* effective potential for $\varphi = 0$, since we shall be interested in the unbroken phase (i.e. $T \geq T_c$). To calculate the second derivative of $\overline{V}_{eff}$ with respect to φ, we remark from equations (19, 20) that $\overline{\Omega}^2$ and $\overline{\Delta}^2$ also depend on φ according to the relations

$$\frac{d\overline{\Omega}^2}{d\varphi} = \frac{24\lambda\varphi - e^2 I_{-1}^d(\overline{\Delta})\frac{d\overline{\Delta}^2}{d\varphi}}{1 + 6\lambda I_{-1}^d(\overline{\Omega})}, \tag{22}$$

$$\frac{d\overline{\Delta}^2}{d\varphi} = 2e^2\varphi - \frac{1}{2}e^2 I_{-1}^d(\overline{\Delta})\frac{d\overline{\Omega}^2}{d\varphi}, \tag{23}$$

where

$$I_{-1}^d(M) = 2\int \frac{d^dk}{(2\pi)^d}\frac{1}{(k^2+M^2)^2}. \tag{24}$$

Replacing equation (23) in (22) we get,

$$\frac{d\overline{\Omega}^2}{d\varphi} = \frac{\left[24\lambda - 2e^4 I_{-1}^d(\overline{\Delta})\right]\varphi}{1 + \left[6\lambda - \frac{1}{2}e^4 I_{-1}^d(\overline{\Omega})\right] I_{-1}^d(\overline{\Omega})}, \tag{25}$$

and the second derivative of the GEP with respect to φ is given by,

$$\frac{d^2\overline{V}_{eff}}{d\varphi^2} = m_0^2 + 12\lambda\varphi^2 + 12\lambda I_0^d(\overline{\Omega}) + 2e^2 I_0^d(\overline{\Delta}) + 2e^4\varphi^2 I_{-1}^d(\overline{\Delta}) - \frac{\left[6\lambda + \frac{1}{2}e^4 I_{-1}^d(\overline{\Delta})\right]\left[24\lambda - 2e^4 I_{-1}^d(\overline{\Delta})\right]\varphi^2}{1 + \left[6\lambda - \frac{1}{2}e^4 I_{-1}(\overline{\Omega})\right] I_{-1}^d(\overline{\Omega})}. \tag{26}$$

Thus we have the formula for the Gaussian mass,

$$\overline{m}^2 \equiv \left.\frac{d^2 V_{eff}}{d\varphi^2}\right|_{\varphi=0} = m_0^2 + 12\lambda I_0^d(\overline{\Omega}_0) + 2e^2 I_0^d(\overline{\Delta}_0), \tag{27}$$

where $\overline{\Omega}_0$ and $\overline{\Delta}_0$ are respectively solutions for $\overline{\Omega}$ and $\overline{\Delta}$ at $\varphi = 0$, explicitly,

$$\overline{\Omega}_0^2 = m_0^2 + 12\lambda I_0^d(\overline{\Omega}_0) + 2e^2 I_0^d(\overline{\Delta}_0), \tag{28}$$

$$\overline{\Delta}_0^2 = e^2 I_0^d(\overline{\Omega}_0). \tag{29}$$

Therefore, from equations (27, 28) we get simply,

$$\overline{m}^2 = \overline{\Omega}_0^2. \tag{30}$$

Hence, we see from the gap equation (27) that the Gaussian mass obeys a generalized "Gaussian" Dyson-Schwinger equation,

$$\overline{m}^2 = m_0^2 + 12\lambda I_0^d(\overline{m}) + 2e^2 I_0^d\left(\sqrt{e^2 I_0^d(\overline{m})}\right). \tag{31}$$

This expression will be used later to describe the system in the neighbourhood of criticality. We also notice that along the lines of [23] we can use equations (27, 28) in order to eliminate m_0 from the gap equations (19, 20) getting the following expression for the gap equation, in the three dimensional case,

$$(x-1) + kL_1(x) - k\Phi^2 = -qL_1(y). \tag{32}$$

In the above equation we have used the following definitions: $\Phi^2 = \frac{4\pi\phi^2}{\overline{m}}$, $x = \frac{\overline{\Omega}^2}{\overline{m}^2}$, $y = \frac{\overline{\Delta}^2}{\overline{\Delta_0}^2}$, $k = 3\frac{\lambda}{\pi\overline{m}}$, $q = \frac{e^2\overline{\Delta_0}}{2\pi\overline{m}}$, $\frac{\overline{m}L_1(x)}{4\pi} = I_0(\overline{m}) - I_0(\overline{\Omega})$ and $\frac{\overline{\Delta_0}L_1(y)}{4\pi} = I_0(\overline{\Delta_0}) - I_0(\overline{\Delta})$. For the three dimensional case $L_1(x) = \sqrt{x} - 1$ which, as in the scalar case [23], allows an explicit solution of equation (32).

In Figures 1, 2 and 3 we illustrate the relation given by equation (32) between x and y, keeping in each figure some kind of constraints for the parameters k, q and Φ. In Figure 1, q and Φ are set to 1, while y is evaluated at different values. It is obtained calculating x for each given value of y, and it can be interpreted as a kind of a level curve in the "normalized mass space". Since we have interest in non-negative mass solutions of equation (32), notice that the positivity of solutions happens when k increases

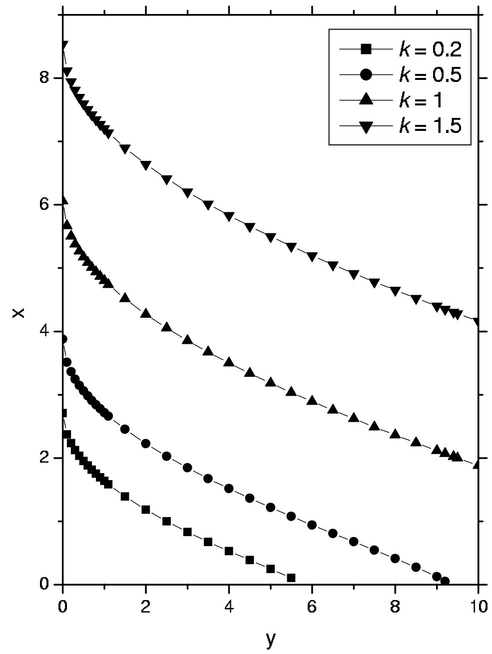

Fig. 1. Plot of $x \times y$ related in the gap equation written in the form of equation (32), with different values of k. q and Φ are set to 1.

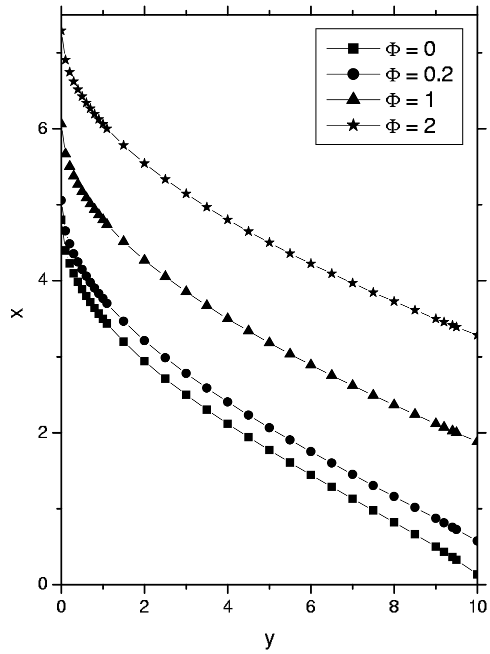

Fig. 3. Plot of $x \times y$ related in the gap equation written in the form of equation (32), with different values of Φ. k and q are set to 1.

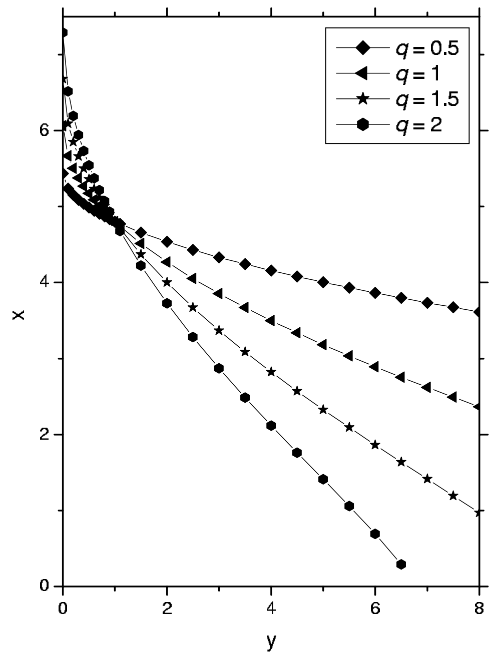

Fig. 2. Plot of $x \times y$ related in the gap equation written in the form of equation (32), with different values of q. k and Φ are set to 1.

and is also non-negative. This fact suggests the confirmation that the coupling constant must be positive-definite. An analogous procedure is done in Figure 2, in which k and Φ are 1. We see that when q tends to zero, we restore the scalar field gap equation. It can be produced in two ways: by the absence of the gauge field ($e = 0$) or the null solution of the gap equation of the gauge field, $\overline{\Delta_0}$. On the other hand, the increasing of q forces x to exist in both positive and negative region. Figure 3 is considered here for completeness, in which k and q are taken to 1. We observe that the non-negative values of Φ do not cause qualitative changes. This particular case of Φ is the only one that we regard here, since we work on the unbroken phase.

In the next section we can analyze in detail this model at the critical region, in which $\overline{\Omega}$ is approximately zero, taking into account also its confinement.

3 Critical behaviour of the confined Ginzburg-Landau model

3.1 The effect of confinement

Let us now consider the system confined between two parallel planes, normal to the x_d-axis, a distance L apart from one another and use Cartesian coordinates $\mathbf{r} = (x_d, \mathbf{z})$, where $\mathbf{z}$ is a $(d-1)$-dimensional vector, with corresponding momenta $\mathbf{k} = (k_d, \mathbf{q})$, $\mathbf{q}$ being a $(d-1)$ -dimensional vector in momenta space. In this case, the model is supposed to describe a superconducting material in the form of a film. Under these conditions the field $\phi(x_d, \mathbf{z})$ satisfies the condition of confinement along the x_d-axis, $\varphi(x_d = 0, \mathbf{z}) = \varphi(x_d = L, \mathbf{z}) =$ const., and should have a mixed series-integral Fourier expansion of the form,

$$\phi(x_d, \mathbf{z}) = \sum_{n=-\infty}^{\infty} c_n \int d^{d-1}\mathbf{q}\, b(\mathbf{q}) e^{-i\omega_n x_d\, -i\mathbf{q}\cdot\mathbf{z}} \tilde{\varphi}(\omega_n, \mathbf{q}), \tag{33}$$

where $\omega_n = 2\pi n/L$ and the coefficients c_n and $b(\mathbf{q})$ correspond respectively to the Fourier series representation over x_d and to the Fourier integral representation over the $(d-1)$-dimensional $\mathbf{z}$-space. The above conditions of confinement of the x_d-dependence of the field to a segment of length L, allow us to proceed with respect to the x_d-coordinate, in a manner analogous as it is done in the imaginary-time Matsubara formalism in field theory. The Feynman rules should be modified following the prescription,

$$\int \frac{dk_d}{2\pi} \rightarrow \frac{1}{L}\sum_{n=-\infty}^{+\infty}, \qquad k_d \rightarrow \frac{2n\pi}{L} \equiv \omega_n. \tag{34}$$

We emphasize however, that here we are considering an Euclidean field theory in d *purely* spatial dimensions, we are *not* working in the framework of finite temperature field theory. Temperature is introduced in the mass term of the Hamiltonian by means of the usual Ginzburg-Landau prescription.

For our purposes we only need the calculation of the integral given in equation (15) in the situation of confinement of the present section. With the prescription (34), the equation corresponding to equation (15) for the confined system can be written in the form,

$$I_0^d(M) = \frac{1}{4\pi^2 L}\sum_{n=-\infty}^{+\infty}\int \frac{d^{d-1}q}{q^2 + an^2 + c^2}, \tag{35}$$

where $q_i = k_i/2\pi$, $a = 1/L^2$ and $c^2 = M^2/4\pi^2$.

Equation (35) can be treated within the framework of the formalism developed in references [28,29]. Using a well known regularization formula [31], we can write equation (35) in the form

$$I_0^d(M) = \frac{\sqrt{a}}{4\pi^{2-d/2}}\Gamma\left(2-\frac{d}{2}\right)A_1^{c^2}\left(\frac{3-d}{2}, a\right), \tag{36}$$

where $A_1^{c^2}\left(\frac{3-d}{2}, a\right)$ is one of the Epstein-Hurwitz zeta-functions, defined by [32]

$$A_K^{c^2}(\nu;\{a_i\}) = \sum_{n_1,\ldots,n_K=-\infty}^{+\infty}(a_1 n_1^2 + \ldots + a_K n_K^2 + c^2)^{-\nu}, \tag{37}$$

with $\mathrm{Re}(\nu) > K/2$ (in our case $\mathrm{Re}(d) < 2$). The Epstein-Hurwitz *zeta*-function can be extended as a meromorphic function to the whole complex ν-plane (for us, to all values of the dimension d), and we obtain after some rather long but straightforward manipulations described in detail in [28], the expression,

$$I_0^d(M) = 2^{-\frac{d}{2}}\pi^{1-\frac{d}{2}}\left[2^{1-\frac{d}{2}}\Gamma\left(1-\frac{d}{2}\right)M^{-2+d} + 2\sum_{n=1}^{\infty}\left(\frac{M}{nL}\right)^{-1+\frac{d}{2}}K_{-1+\frac{d}{2}}(MLn)\right], \tag{38}$$

where K_ν are the Bessel functions of third kind.

3.2 Critical behaviour

Now we are able to analyze the critical behaviour of the system under consideration. It is important to say that in this context we will deal with the system obeying the following condition [18]:

$$\xi(T) \equiv \overline{m}^{\,-1} \gg \lambda(T) \gg L, \tag{39}$$

where $\xi(T)$ and $\lambda(T)$ are the GL correlation and penetration lengths, defined by

$$\xi(T) = \frac{\xi_0}{|t|^{1/2}},\ \ \lambda(T) = \frac{\lambda_0}{|t|^{1/2}};\ t = \frac{T-T_c}{T_c}, \tag{40}$$

being T_c the transition temperature of the film, ξ_0 and λ_0 the intrinsic coherence and penetration length, respectively (see next section). Therefore, here we will work only with type-I superconducting films, because equation (39) holds for type-I materials at the neighborhood of the criticality. Thus, we restrict our study to the normal-supercontucting state transition, in which the last state obeys the Meissner law.

After that, now we can take $M = \overline{m}$ in equation (38) and let us restrict ourselves to the neighbourhood of criticality, that is, to the region defined by $\overline{m} \approx 0$. Then the asymptotic formula,

$$K_\nu(z) \approx \frac{1}{2}\Gamma(\nu)\left(\frac{z}{2}\right)^{-\nu}, \quad (z \sim 0) \tag{41}$$

allows to write equation (38) in the form

$$I_0^d(\overline{m} \approx 0) \approx \frac{\pi^{1-\frac{d}{2}}}{2}\Gamma\left(1-\frac{d}{2}\right)\frac{1}{L^{d-2}}\zeta(d-2), \tag{42}$$

where $\zeta(d-2)$ is the Riemann *zeta*-function, $\zeta(d-2) = \sum_{n=1}^{\infty}(1/n^{d-2})$, defined for $d > 3$. For $d \gtrsim 3$, in the sense of the analytic continuation in dimension of ζ-functions, we obtain the expression,

$$I_0^d(\overline{m} \approx 0) \approx \frac{1}{2\sqrt{\pi}}\frac{1}{L}\zeta(d-2). \tag{43}$$

The integral $I_0^d\left(\overline{\Delta}_0 = \sqrt{e^2 I_0^d(\overline{m})}\right)$, which enters equation (31), must be considered carefully. For a dimension $d \gtrsim 3$, we get,

$$I_0^d(\overline{\Delta}_0) \approx 2^{-\frac{3}{2}}\pi^{-\frac{1}{2}}\left[2^{-\frac{1}{2}}\Gamma\left(-\frac{1}{2}\right)\overline{\Delta}_0 + 2\sum_{n=1}^{\infty}\left(\frac{\overline{\Delta}_0}{nL}\right)^{\frac{1}{2}}K_{\frac{1}{2}}(\overline{\Delta}_0 Ln)\right], \tag{44}$$

or, using the exact expression for the summation in the above equation,

$$\sum_{n=1}^{\infty}\left(\frac{\overline{\Delta}_0}{nL}\right)^{\frac{1}{2}}K_{\frac{1}{2}}(\overline{\Delta}_0 Ln) = -\sqrt{\frac{\pi}{2}}\frac{1}{L}\ln\left(1-e^{-\overline{\Delta}_0 L}\right), \tag{45}$$

Equation (44) becomes (with $\overline{\Delta}_0 = \sqrt{e^2 I_0^d(\overline{m})}$)

$$I_0^d\left(\sqrt{e^2 I_0^d(\overline{m})}\right) \approx \frac{1}{2\sqrt{\pi}}\left[\frac{1}{\sqrt{2}}\Gamma\left(-\frac{1}{2}\right)\sqrt{e^2 I_0^d(\overline{m})} -\sqrt{2\pi}\frac{1}{L}\ln\left(1-e^{-\sqrt{e^2 I_0^d(\overline{m})}L}\right)\right]. \quad (46)$$

However, notice that if the are in the limit $\overline{m} \approx 0$, we see replacing $I_0^d(\overline{m})$ from equation (43) in the exponential, that the logarithm in the last term of equation (46) will disappear at $d = 3$, due to the divergence of $\zeta(d-2)$ as $d \to 3$. Hence, equation (46) becomes simply, for a dimension $d \gtrsim 3$,

$$I_0^d\left(\sqrt{e^2 I_0^d(\overline{m} \approx 0)}\right) \approx \frac{e}{2\pi^{1/4}\sqrt{2}}\frac{1}{L^{\frac{1}{2}}}\zeta^{\frac{1}{2}}(d-2). \quad (47)$$

Thus we can write the Gaussian gap equation (31) in the neighbourhood of criticality in the form,

$$\overline{m}^2 \approx m_0^2 + \frac{24}{\sqrt{\pi}}\lambda\frac{1}{L}\zeta(d-2) - \frac{1}{\pi^{1/4}\sqrt{2}}e^3\frac{1}{L^{\frac{1}{2}}}\zeta^{\frac{1}{2}}(d-2). \quad (48)$$

For $\overline{m} = 0$, equation (48) defines a critical equation for $d \gtrsim 3$. But it is well known that the *only* singularity of the *zeta*-function $\zeta(z)$ is a pole at $z = 1$, which makes equation (48) meaningless as it stands for $d = 3$, just the physically interesting situation.

However, we can give a physical sense to equation (48) for $d = 3$, by means of a renormalization procedure. This can be done using the formula,

$$\lim_{z\to 1}\left[\zeta(z) - \frac{1}{1-z}\right] = \gamma, \quad (49)$$

where γ is the Euler constant, to define for $d \gtrsim 3$ a new, *renormalized* mass m, related to the former bare mass by,

$$m^2 = m_0^2 - \frac{24\lambda}{\sqrt{\pi}L(d-3)} + \frac{e^3}{2\pi^{1/4}\sqrt{2L}}\sum_{p=1}^{\infty} C_{\frac{1}{2}}^{p}\gamma^{\frac{1}{2}-p}\frac{(-1)^p}{(d-3)^p}, \quad (50)$$

where $C_{\frac{1}{2}}^{p}$ are appropriate generalizations of the coefficients of the binomial expansion for a fractional power, notice that substracting this pole preserves the sign behaviour with the temperature of the ϕ^2 coefficient in equation (4), a similar procedure has been done in a more implicit way for the finite temperature Gaussian effective potential [25]. Then replacing the above equation in equation (48) and using the binomial formula to expand $\zeta^{1/2}(d-2) \approx [\gamma - (1/(d-3))]^{1/2}$ we obtain, for $d = 3$,

$$\overline{m}^2 \approx m^2 + \frac{24\gamma\lambda}{\sqrt{\pi}}\frac{1}{L} - \frac{e^3}{\pi^{1/4}\sqrt{2}}\frac{\gamma^{\frac{1}{2}}}{L^{\frac{1}{2}}}. \quad (51)$$

Taking $m^2 = a(T/T_0 - 1)$, with $a > 0$ we have from the above equation for $\overline{m}^2 = 0$, the critical temperature as a function of the film thickness L and of the bulk transition temperature, T_0,

$$T_c(L) = T_0\left[1 - \frac{24\gamma\lambda}{a\sqrt{\pi}}\frac{1}{L} + \frac{e^3\sqrt{\gamma}}{a\pi^{1/4}\sqrt{2}}\frac{1}{\sqrt{L}}\right]. \quad (52)$$

This equation describes the behaviour of the critical temperature of a type-I superconducting film as a function of its thickness L, taking into account the gauge fluctuations. Of course when $L \to \infty$, we recover the critical temperature of the material in bulk form. We see clearly two separated contributions in the expression for the critical temperature in equation (52). The first one due to the self-interaction of the scalar field, and the other coming from the interaction between the scalar and gauge fields. This last one would characterize a phase transition considering the intrinsic magnetic fluctuations due to the Cooper pairs. It should be noticed that the self interaction contribution to the critical temperature depends on the inverse of the film thickness, while the charged contribution goes with the inverse of the square root of L.

From equation (52) we see that the transition temperature vanishes for a value of L given by,

$$L^{(0)} = \left[\sqrt{\frac{\gamma}{2\sqrt{\pi}}}\frac{e^3}{2a} - \left(\frac{\gamma e^6}{8a^2\sqrt{\pi}} + \frac{24\gamma\lambda}{a\sqrt{\pi}}\right)^{\frac{1}{2}}\right]^2. \quad (53)$$

For $L < L^{(0)}$, the critical temperature (in absolute units) becomes negative, which means that $L^{(0)}$ is the minimal physically allowed film thickness, below which the superconducting transition is suppressed.

4 Comparison with experimental results

Our approach, based only on the GL model, gives a functional form for the decrease of the transition temperature with the inverse film thickness which agrees qualitatively with experimental observations [9–13,15–17]. These experimental results exhibit a *linear* decrease of the critical temperature with the inverse of the film thickness for several kinds of materials. Also we have obtained a minimal film thickness below which the critical temperature becomes negative. This can be interpreted as the minimal film thickness below which the transition is suppressed *in the context* of the GL model. It gives a limit of validity for $T_c(L)$ in equation (52) in the context of the proposed model. Moreover, equation (39) restricts our model to the type-I superconducting films. In particular we can not see any overlap with a two-dimensional Kosterlitz-Thouless transition, due to the presence of the gauge field fluctuations (in addition to the restriction of looking at only type-I materials).

In order to compare more precisely our results with experimental ones, we still need to estimate the magnitude of $L^{(0)}$ for films made of a particular material. For that we start showing the compatibility of the quantities of our field theoretical model with the phenomenological parameters. In equation (52), the parameters λ (coupling constant of the ϕ^4 term) and e^2 (squared charge accounting for effects due to gauge fluctuations) have mass dimension, and a has squared mass dimension (let us remind that we have used in Natural Units, $c = \hbar = k_B = 1$). To proceed, we shall restore the SI units, according to reference [30]: we remember that a factor $1/k_B T_0$ is implicit in the exponent of equation (3) and we rescale the fields and coordinates by $\phi \to \phi_{new} = \sqrt{\xi_0/k_B T_0}\phi$, $A \to A_{new} = \sqrt{\xi_0/k_B T_0}A$ and $x \to x_{new} = x/\xi_0$, with $\xi_0 = 0.18\hbar v_F/k_B T_0$, being the intrinsic coherence length (v_F is the Fermi velocity). Therefore a, λ and e become dimensionless, in such a way that we have these parameters related to the well known 3-dimensional phenomenological Ginzburg-Landau parameters a, g and q by [30]

$$a = 1,\ \lambda \approx 111.08\left(\frac{T_0}{T_F}\right)^2,\ e \approx 2.59\sqrt{\frac{\alpha v_F}{c}}, \qquad (54)$$

where T_F and α are respectively the Fermi temperature and the fine structure constant. To take into account these changes, the thickness L in equation (52) must be rescaled by $L \to L_{new} = L/\xi_0$. Thus, the replacement of equations (54) in (52) and (53) yields the critical temperature and the minimal thickness in terms of parameters directly related to the characteristic quantities of materials.

Notice, however, that we have considered a perfect film, without impurities. If we consider that the type-I film contain impurities, the intrinsic coherence length and the coupling constants must be changed. Then they become $\xi_0 \to r^{1/2}\xi_0$, $\lambda \to 2r^{-3/2}\lambda$ and $e \to r^{1/4}e$, where $r \sim 0.18C^{-1}$, with $C = \xi_0/l$, l being the mean free path of the electron. The existence of the factor 2 in λ will be discussed later. The expression for r is valid in the limit of ξ_0 reasonably greater than l. It is also worth to note that we will regard that the impurities in the film are not sufficient to remove it from the type-I region, i.e. the film is made of a type-I material in a such way that it is not quite dirty to put it in the type-II limit. Hence, it continues having a usual normal-superconducting phase transition. Thus, equation (52) is rewritten as

$$T_c = T_0\left[1 - \frac{9646.2C\xi_0 t_{0F}^2}{L} + \frac{7.87\times 10^{-4}\left(\xi_0 v_{Fc}^3\right)^{\frac{1}{2}}}{C\sqrt{L}}\right], \qquad (55)$$

where $t_{0F} = T_0/T_F$ and $v_{Fc} = v_F/c$. In the same way, which the assumptions given above, equation (53) becomes

$$L^{(0)} = \xi_0\left(\frac{3.94\ \times 10^{-4}v_{Fc}^{\frac{3}{2}}}{C} - \left[\frac{1.54\times 10^{-7}v_{Fc}^3}{C^2} + 9646.2Ct_{0F}^2\right]^{\frac{1}{2}}\right)^2. \qquad (56)$$

Let us now consider a superconducting film made from niobium, characterized by $v_F = 1.37 \times 10^6$ m/s, $T_0 = 9.3$ K and $T_F = 6.18 \times 10^4$ K. We also chose $C \sim 100$. With these data, the second coefficient between brackets in equation (55) has a magnitude 10^4 greater than the third coefficient, and since L ranges in the order of 10–100 angstrom, the term due the contribution of gauge fluctuations is relatively small. From equation (56) we obtain an estimate for the minimal allowed thickness for the existence of a normal-superconducting phase transition in Nb films,

$$L^{(0)}_{C\sim 100}(\text{Nb}) \approx 44\ \text{Å}. \qquad (57)$$

If we consider $C \sim 200$ and $T_0 = 8.4$ K we obtain from equation (56)

$$L^{(0)}_{C\sim 200}(\text{Nb}) \approx 80\ \text{Å}. \qquad (58)$$

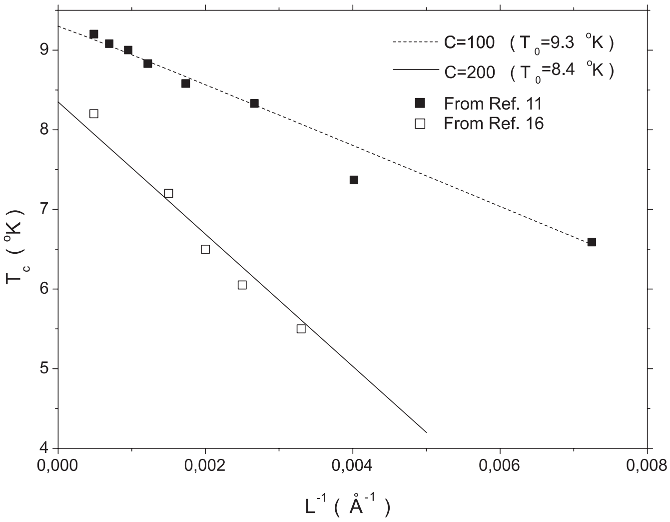

Fig. 4. Plot of the transition temperature T_c as defined by equation (55), taking $C \sim 100$ and $C \sim 200$. Data are obtained from references [11,16].

In what the behaviour of the critical temperature with the film thickness is concerned, in Figure 4 we plot equation (55) for Nb in two cases: (i) for a film (upper curve) whose the relevant parameters take the values $C \sim 100$, $T_0 = 9.3$ K, and (ii) for a film (lower curve) with $C \sim 200$ and $T_0 = 8.4$ K. We see that in both cases we have a good agreement with the experimental results from respectively references [11,16].

It should be noticed that the choice of the parameter $C \sim 100$ was done in a *heuristic* and reasonable manner to agree with the results of [11], as well as $C \sim 200$ which is in accordance with [16]. In this context, the second example is more disordered and dirty than the first one, and therefore C is relatively greater. Besides, the factor 2 that arose in the definition of λ in the dirty limit can be understood, according to [16], due to the decreasing of a factor 2 in the density of states at the Fermi energy $N(0)$ for disordered films, and since λ is conversely proportional to $N(0)$, this factor then appears.

We believe that our approach is an alternative way to introduce phenomenologically, via the compactified GL model, corrections that take into account microscopic effects that arise in type-I superconducting films, such as proximity, localization and an increased residual resistivity.

Finally, we remark that the use of the term "minimal thickness" is constrained to the non-observation of a usual normal-superconducting phase transition below $L^{(0)}$. It means that, even in the case $e = 0$ in equation (52), the Kosterlitz-Thouless transition is not captured, since we cannot take into account vortices in our model. However, the existence of $L^{(0)}$ in this scenario would be interpreted as a crossover to the bidimensional case, in which the Kosterlitz-Thouless transition is kept if vortices are respected.

5 Conclusions

In this paper we have considered the Ginzburg-Landau model, confined between two parallel planes, and in the transverse unitarity gauge, as a model to describe a type-I superconducting film. To generate the contributions from gauge fluctuations, we have used the Gaussian effective potential [23–27], which allows to obtain a gap equation that can be treated with the method of recent developments [28,29]. We have deduced a critical equation that describes the changes in the critical temperature T_c due to confinement. Independent contributions from the self interaction of the scalar field and from the gauge field fluctuations are found. Our approach suggests a minimal film thickness for superconducting transitions, with or without the presence of gauge interactions. This can be clearly seen from equation (52), where a linear decreasing of the critical temperature with the inverse of film thickness is recovered when we take $e = 0$. Even though, with $e \neq 0$ a line very near the linear decreasing of T_c with $1/L$ can be directly obtained from equation (55), because the term due to gauge effects fluctuations is very small compared to the term generated from self coupling. Our results are in qualitative agreement with the behaviour that has been found *experimentally* in materials containing transition metals, for example, in Pb [9], in W-Re alloys [10], in Nb [11,13,15,16], Mo-Ge [12] and in epitaxial MgB_2 films [17]. We would like to emphasize that the linear character of the decreasing of the transition temperature obtained in this paper, emerge solely as a topological effect of the compactification of the Ginzburg-Landau model in one direction. We remark that we have calculated a minimal thickness for a film made from Nb and find that the results are in good agreement with recent experimental data [11,16]. Finally, it should be observed that our restriction to work only with type-I films allows us to analyze the usual normal-superconducting state transition, and thus we do not consider other universality classes of phase transitions, like the Kosterlitz-Thouless transition, which is important in the framework of two-dimensional systems and considers the unbinding of vortex pairs. This is an important phenomenon and we expect in the future to be able to extend the present analysis in order to include vortex effects in quasi-two-dimensional systems.

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